Errata of "Unifying Splitting"

Gabriel Ebner, Jasmin Blanchette, Sophie Tourret

Soundness of $\rhd_{Red_{\mathrm{F}}}$

In the paper, just before sect. 2.3, we have a bit quickly stated that $\rhd_{Red_{\mathrm{F}}}$ is *sound* if $M \rhd_{Red_{\mathrm{F}}} N$ entails $M \models\!\!\approx N$. In fact, due to our entailment being disjunctive, this would make the simplification rules that just delete formulas without replacement unsound. This is not the expected behaviour of the soundness property (removing formulas is a sound operation since it cannot falsify a satisfiable formula). The correct statement of soundness for transitions such as $\rhd_{Red_{\mathrm{F}}}$ is: $\forall C \in N.\ M \models\!\!\approx \{C\}$. This definition is compatible with both conjunctive and disjunctive entailments. For conjunctive entailments, it coincides with $M \models\!\!\approx N$.

Axiom (D6)

The axiom (D6) introduced at the beginning of Sect. 3 should be $\emptyset \not\models \emptyset$ rather than $\emptyset \not\models\!\!\approx \emptyset$. To reflect this, $\models$ should be used instead of $\models\!\!\approx$ in the sentence introducing (D6). In practice, this is not restrictive because it only prevents $\models$ to be the trivial always true relation.

COLLECT rule side-condition

The side-condition of the COLLECT rule should be $\{\bot \leftarrow A_i\}_{i=1}^{n} \models \{\bot \leftarrow A\}$ instead of $\{\bot \leftarrow A_i\}_{i=1}^{n} \models\!\!\approx \{\bot \leftarrow A\}$. The other side condition of COLLECT ($C \neq \bot$) remains unchanged.

Note: the "Axiom (D6)" and "COLLECT rule side-condition" errata were both motivated by an issue in the proof of Th. 19 to prove that COLLECT is a simplification rule. With the original version, it is not possible to derive $\emptyset \models\!\!\approx \{\bot\}$ as needed but only $\mathit{fml}(\mathcal{J}) \models\!\!\approx \emptyset$ for all $\mathcal{J}$. With the corrected version $\emptyset \models \{\bot\}$ can be derived and ensures a contradiction with (D6).

TRIM rule side-condition

For the TRIM rule to be proven a simplification rule in Th. 19 (Simplification) as described in the paper, B must be a strict subset of $A \cup B$. In practice, we restrict the use of this rule even more, by imposing $A \cap B = \emptyset$ and $A \neq \emptyset$ as side-conditions.

Precision in the statement of Th. 14 (Soundness)

For inference rules being sound is at the level of inferences, i.e. ι is sound if $\mathit{prems}(\iota) \models\!\!\approx \{\mathit{concl}(\iota)\}$. However, for simplification rules, that directly operate at the level of transitions (they are not inference rules), soundness refers to the soundness of $\rhd_{Red_{\mathrm{F}}}$ as stated in these errata.

Case COLLECT in the proof of Th. 14

The rule COLLECT is trivially sound (no new formula added). The argument in the proof of Th.14 on COLLECT regards completeness, not soundness, and can thus be completely ignored.

Case TRIM in the proof of Th. 14 (Soundness)

To prove that the rule TRIM is sound, we must show that $\{\bot \leftarrow A_i\}_{i=1}^{n} \cup \{C \leftarrow A \cup B\} \approx\models \{C \leftarrow B\}$. This amounts to proving that for all $\mathcal{J}$ such that $B \subseteq \mathcal{J}$, $\mathit{fml}(\mathcal{J}) \cup (\{\bot \leftarrow A_i\}_{i=1}^{n})_{\mathcal{J}} \cup \{C \leftarrow A\}_{\mathcal{J}} \approx\models C$.

Let us consider a $\mathcal{J}$ such that $B \subseteq \mathcal{J}$. By the TRIM rule constraints, $\mathit{fml}(\mathcal{J}) \cup (\{\bot \leftarrow A_i\}_{i=1}^{n})_{\mathcal{J}} \cup \{\bot \leftarrow A\}_{\mathcal{J}} \approx\models \{\bot\}$. This means that either $A \subseteq \mathcal{J}$, or there exists an $i \in \{1..n\}$ such that $A_i \subseteq \mathcal{J}$, or $\mathit{fml}(\mathcal{J}) \approx\models \{\bot\}$. If $A \subseteq \mathcal{J}$ then $\mathcal{J}$ enables $\{C \leftarrow A\}$ thus $\{C \leftarrow A\}_{\mathcal{J}} = \{C\}$ and by (D2) we know that $\{C\} \approx\models \{C\}$. If $A_i \subseteq \mathcal{J}$ for some $i \in \{1..n\}$, then $(\{\bot \leftarrow A_i\}_{i=1}^{n})_{\mathcal{J}} = \{\bot\}$, and by (D1) $\{\bot\} \approx\models \emptyset$. In the remaining case, we apply (D4) on $\mathit{fml}(\mathcal{J}) \approx\models \{\bot\} \cup \{C\}$ and $\mathit{fml}(\mathcal{J}) \cup \{\bot\} \approx\models \{C\}$ to obtain $\mathit{fml}(\mathcal{J}) \approx\models \{C\}$. The former premise, $\mathit{fml}(\mathcal{J}) \approx\models \{\bot\} \cup \{C\}$, follows by (D3) from the case assumption, and the latter premise follows from (D1) and (D3).

In all cases, the result we need to prove follows by (D3).