Errata of "Superposition with Lambdas"

Alexander Bentkamp, Jasmin Blanchette, Sophie Tourret, Petar Vukmirović, Uwe Waldmann

Nonstrict term order

Employing a nonstrict term order for the Boolean-free λ-superposition calculus renders the calculus incomplete, contrary to what is claimed in the article. The most straightforward way to fix the calculi is to replace the nonstrict term order $\gtrsim$ by the reflexive closure $\succeq$ of the strict term order $\succ$. Alternatively, for all rules using the nonstrict term order $\gtrsim$ after applying a substitution σ, we can instead use the nonstrict term order $\gtrsim$ before applying σ, and add another condition that uses the reflexive closure $\succeq$ of the strict term order after applying σ.

Here is an example demonstrating the incompleteness: Let $b \succ a$. Consider the clause $C = X\,a\,a \not\approx X\,b\,a \vee X\,a\,a \not\approx X\,a\,b$. Clearly, it is unsatisfiable, which can be shown using any instantiation of X that ignores both arguments. The empty clause should be derivable by applying ERes twice, but ERes does not apply, assuming that none of the literals are selected. This is because, for ERes to apply, a literal must be $\gtrsim$-maximal after applying the unifier σ. For the first literal, a most general unifier would be $\sigma = \{X \mapsto \lambda u\,v.\; Y\,v\}$, yielding $C\sigma = Y\,a \not\approx Y\,a \vee Y\,a \not\approx Y\,b$, but the first literal of $C\sigma$ is not $\gtrsim$-maximal. Similarly, for the second literal, a most general unifier would be $\sigma = \{X \mapsto \lambda u\,v.\; Y\,u\}$, yielding $C\sigma = Y\,a \not\approx Y\,b \vee Y\,a \not\approx Y\,a$, but the second literal of $C\sigma$ is not $\gtrsim$-maximal.

The error in the completeness proof lies in how Lemma 50 employs Lemma 49. For Lemma 49, it is cruicial that the substitution σ is fixed because the $\gtrsim$-eligible literal guaranteed by Lemma 49 depends on σ. The proof of Lemma 50, however, would only work if Lemma 49 gave us a single literal that is $\gtrsim$-eligible for all substitutions σ. For instance, for the ERes rule, the proof of Lemma 50 claims that we can assume the literal $s \not\approx s'$ to be the one guaranteed to be $\gtrsim$-eligible by Lemma 49 "without loss of generality". This is not the case because at that point, we have already fixed σ to be the most general unifier of s and s'. The $\gtrsim$-eligible literal guaranteed by Lemma 49 may be a literal other than $s \not\approx s'$, which would yield a different most general unifier.