



max planck institut
informatik

Extending an Isabelle Formalisation of CDCL to Optimising CDCL





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joint work with Christoph Weidenbach

Matryoshka+Veridis 2019

Let's find a model with minimal weight



	10	→		4
	20	→		13

Optimal partial model: 

Optimal total model: →  

How reliable is the theory?

Conference version

Branch and Bound for Boolean Optimization and
the Generation of Optimality Certificates

Javier Larrosa, Robert Nieuwenhuis, Albert Oliveras, and Enric Rodríguez-Carbonell (SAT 2009)

A literal l is *true* in I if $l \in I$, *false* in I if $\neg l \in I$, and *undefined* in I otherwise.

A clause set S is true in I if all its clauses are true in I . Then I is called a *model* of S , and we write $I \models S$ (and similarly if a literal or clause is true in I).

Journal version

A Framework for Certified Boolean Branch-and-Bound Optimization

Javier Larrosa, Robert Nieuwenhuis, Albert Oliveras, and Enric Rodríguez-Carbonell (JAR 2011)

literals of a clause C are false in I . A clause set S is true in I if all its clauses are true in I ; if I is also total, then I is called a *total model* of S , and we write $I \models S$.

How reliable is the theory?

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Automation of Logic (Chapman & Hall/CRC Studies in Informatics) Hardcover – 22. July 2019

by Christoph Weidenbach (Autor)

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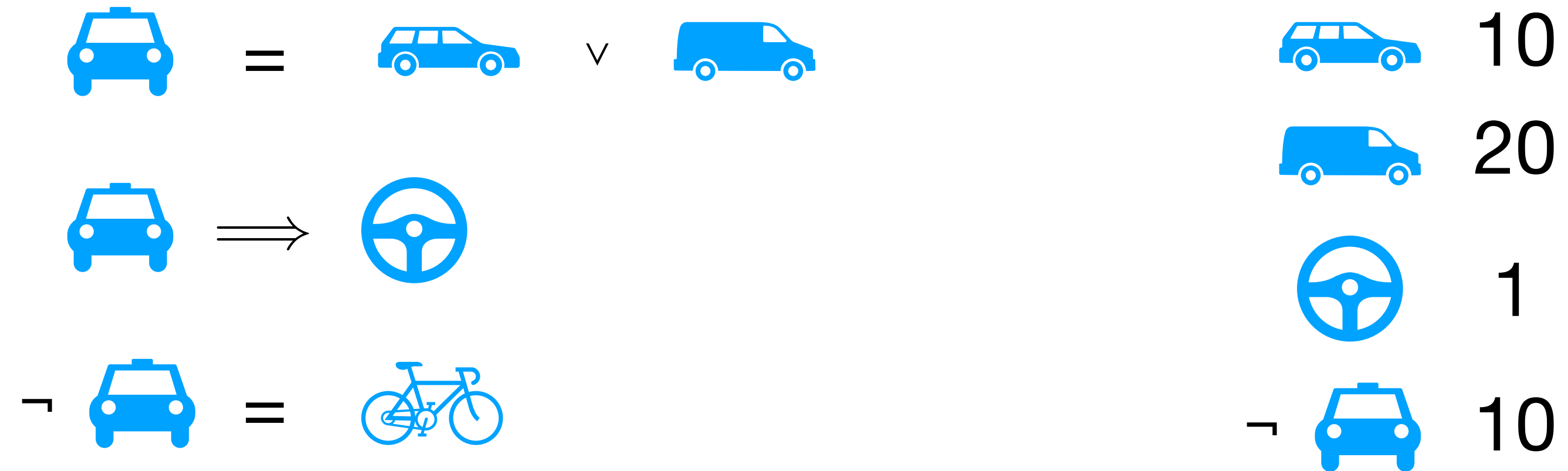
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Lemma 2.15.4. The OCDCL calculus with a reasonable strategy has only 2 normal forms: $(M; N; U; 0; \perp; O)$ where $O \neq \epsilon$, $O \models N$ and $\text{cost}(O)$ is optimal, and $(M; N; U; 0; \perp; \epsilon)$ where N is unsatisfiable.

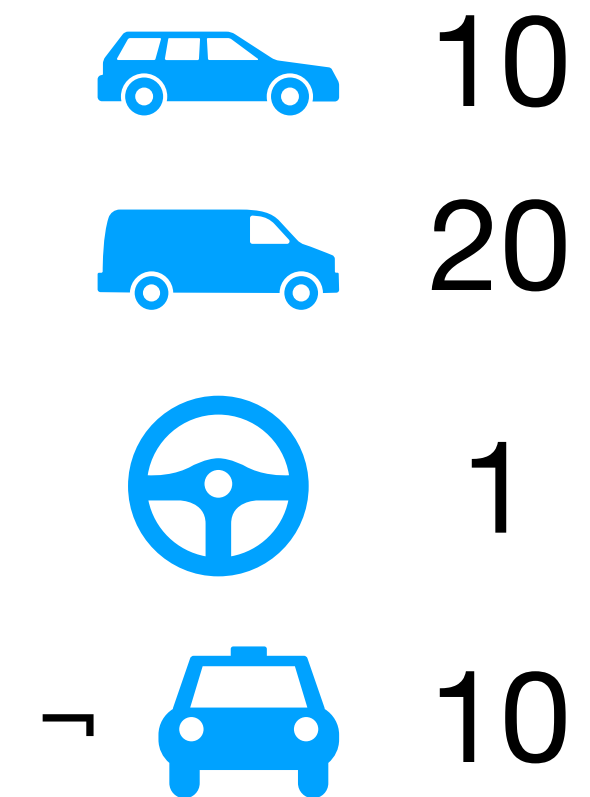
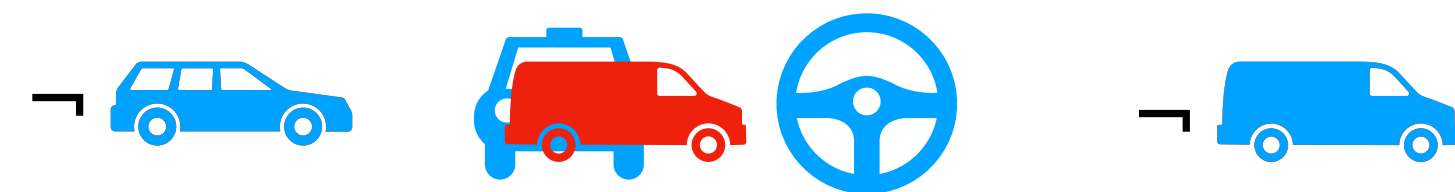
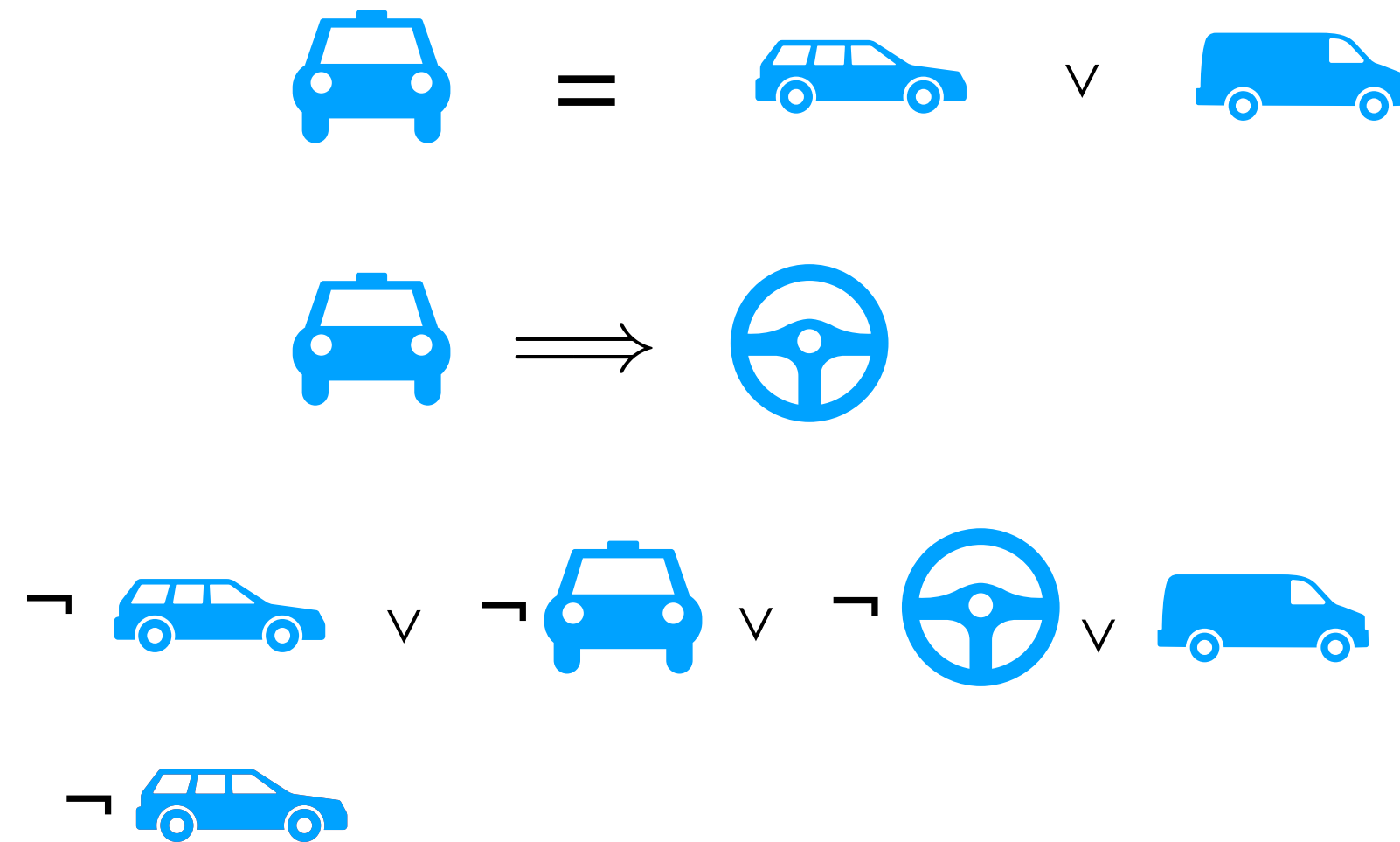
Let's optimise our problem



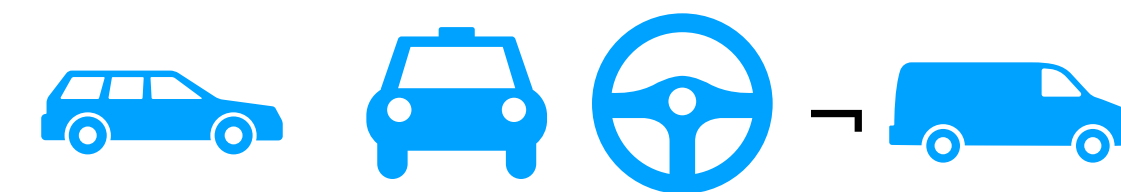
Let's optimise our problem

OCDCL = CDCL + identify better models
+ conflicts based on weights

Let's optimise our problem



Optimal model 11



How lazy do you like your formalisation?

Christoph's view:

$\text{OCDCL}_W = \text{CDCL} + \text{improve} + \text{conflict rules}$

copy-paste of proofs

My first idea:

$\text{OCDCL} = \text{CDCL} + \text{improve} +$
 $\{-M. \text{ cost } M \geq \text{min_cost}\}$

reuse CDCL proofs

How lazy do you like your formalisation?

My first idea:

$$\text{OCDCL} = \text{CDCL} + \text{improve} + \{-M. \text{ cost } M \geq \text{min_cost}\}$$

reuse CDCL proofs

My second idea:

$$\text{CDCL}_{\text{bnb}} = \text{CDCL} + \text{improve} + \mathcal{T}(\text{min_cost})$$

reuse CDCL proofs

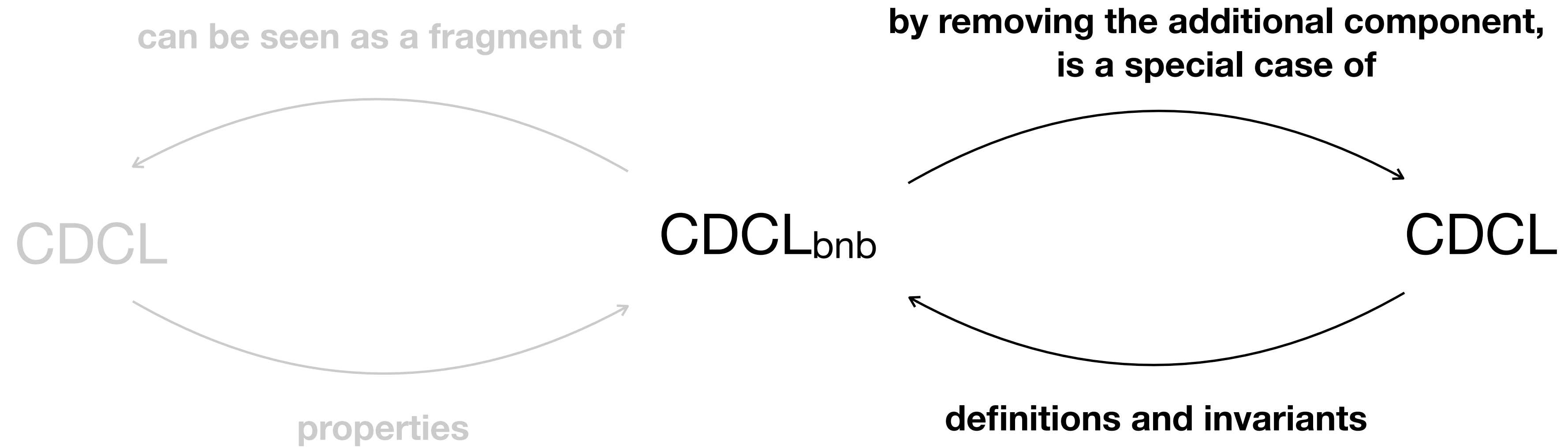
How lazy do you like your formalisation?

$$\text{CDCL}_{\text{bnb}} = \text{CDCL} + \text{improve} + \mathcal{T}(\text{min_cost})$$

$$\text{OCDCL} = \text{CDCL}_{\text{bnb}} \text{ where } \mathcal{T}(\text{min_cost}) = \{-M. \text{cost} \mid M \geq \text{min_cost}\}$$

$$\text{OCDCL}_W = \text{OCDCL} + \text{restrictions}$$

Reuse!



Reuse!

Propagate rule

in Isabelle

$$\begin{aligned} C \vee L \in N \implies M \models_{as} \neg C \implies \text{undefined_lit } M \ L \implies \\ (M, N) \Rightarrow_{\text{CDCL}} (L \# M, N) \end{aligned}$$

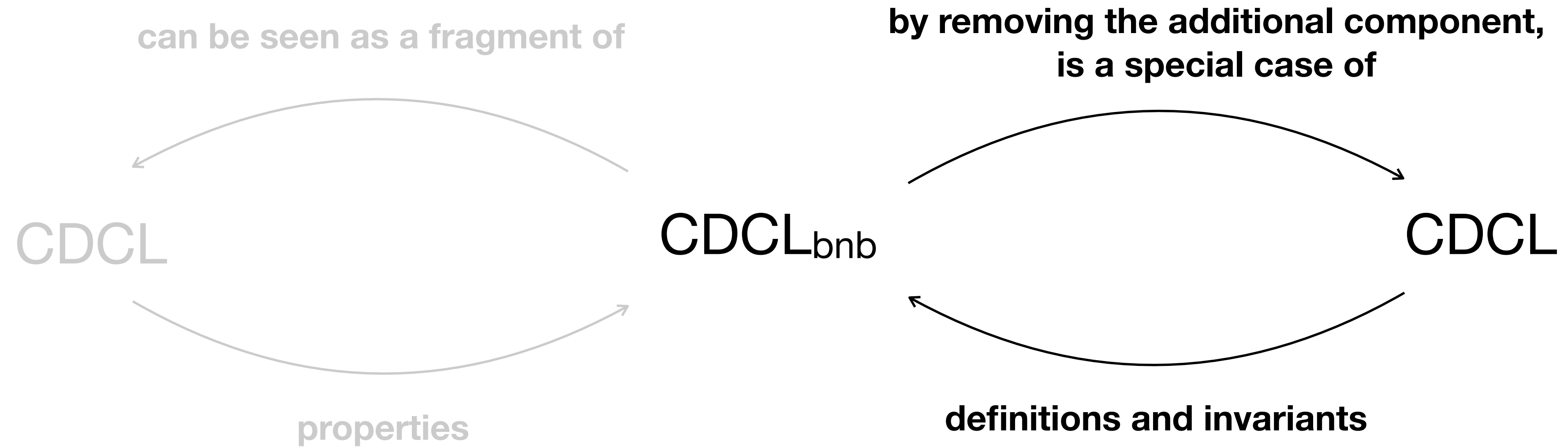
**obtained for free, thanks to abstraction over the state!
also invariants and theorems can be reused**

Propagate rule

in Isabelle

$$\begin{aligned} C \vee L \in N \implies M \models_{as} \neg C \implies \text{undefined_lit } M \ L \implies \\ (M, N, 0) \Rightarrow_{\text{CDCLbnb}} (L \# M, N, 0) \end{aligned}$$

Reuse!



Translate to reuse

Propagate rule

in Isabelle

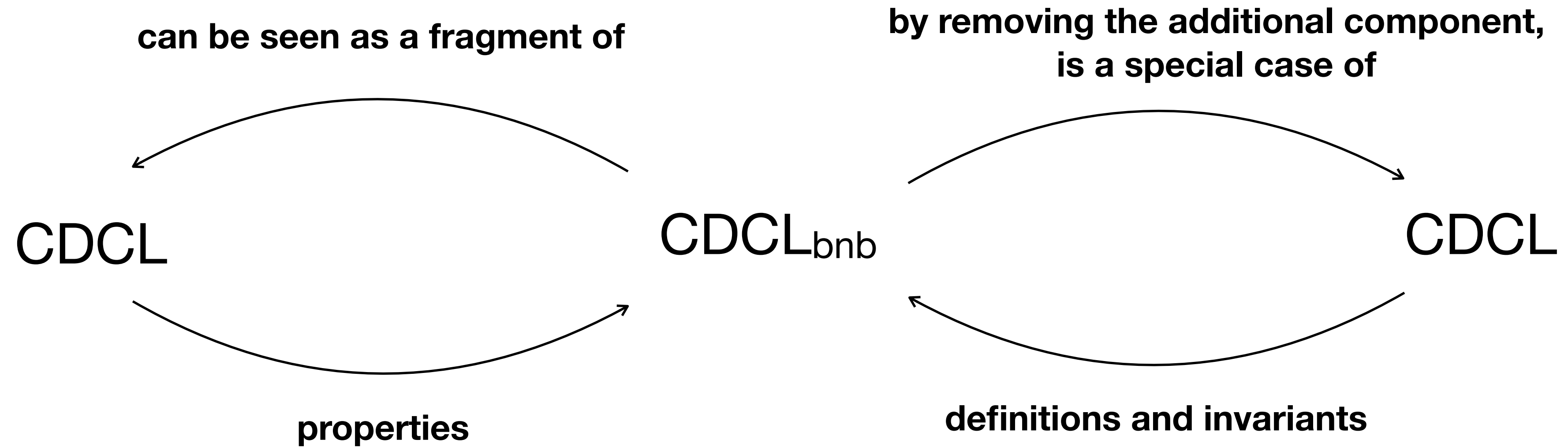
$$C \vee L \in N \implies M \models_{\text{as}} \neg C \implies \text{undefined_lit } M \ L \implies \\ (M, N, 0) \Rightarrow_{\text{CDCLbnb}} (L \# M, N, 0)$$

Propagate rule

in Isabelle

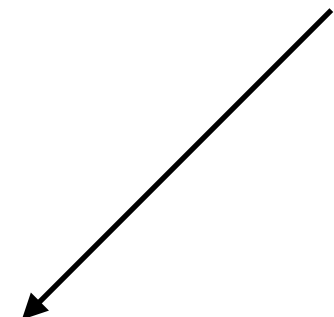
$$C \vee L \in N + \mathcal{I}(\text{min_cost}) \implies M \models_{\text{as}} \neg C \implies \\ \text{undefined_lit } M \ L \implies \\ (M, N + \mathcal{I}(\text{min_cost}), 0) \Rightarrow_{\text{CDCL}} \\ (L \# M, N + \mathcal{I}(\text{min_cost}), 0)$$

Reuse!



Reuse in practise!

ignore the additional
component



$$\text{CDCL}_{\text{bnb}} = \text{CDCL} + \text{improve} + \mathcal{I}(\text{min_cost})$$

Inherited:

Definitions (for free)

Reuse in practise!

no strategy
but terminating

well-founded
for most applications

$$\text{CDCL}_{\text{bnb}} = \text{CDCL} + \text{improve} + \mathcal{T}(\text{min_cost})$$

Inherited:

Termination (for free)

Definitions (for free)

Reuse in practise!

CDCL_{bnb} does not know anything about what is optimised!

Inherited:

Termination (for free) Definitions (for free)

Ends with an unsat set (nearly for free)

Why does it work?

OCDCL = CDCL_{bnb} where

$$\mathcal{T}(\text{min_cost}) = \{-M. \text{ cost } M \geq \text{min_cost}\}$$

Lemma

If I is a total model of N with $\text{cost} < \text{min_cost}$,
then I is a model of $N + \mathcal{T}(\text{min_cost})$

Why does it work?

Lemma

If I is a total model of N with $\text{cost} < \text{min_cost}$,
then I is a model of $N + \mathcal{T}(\text{min_cost})$

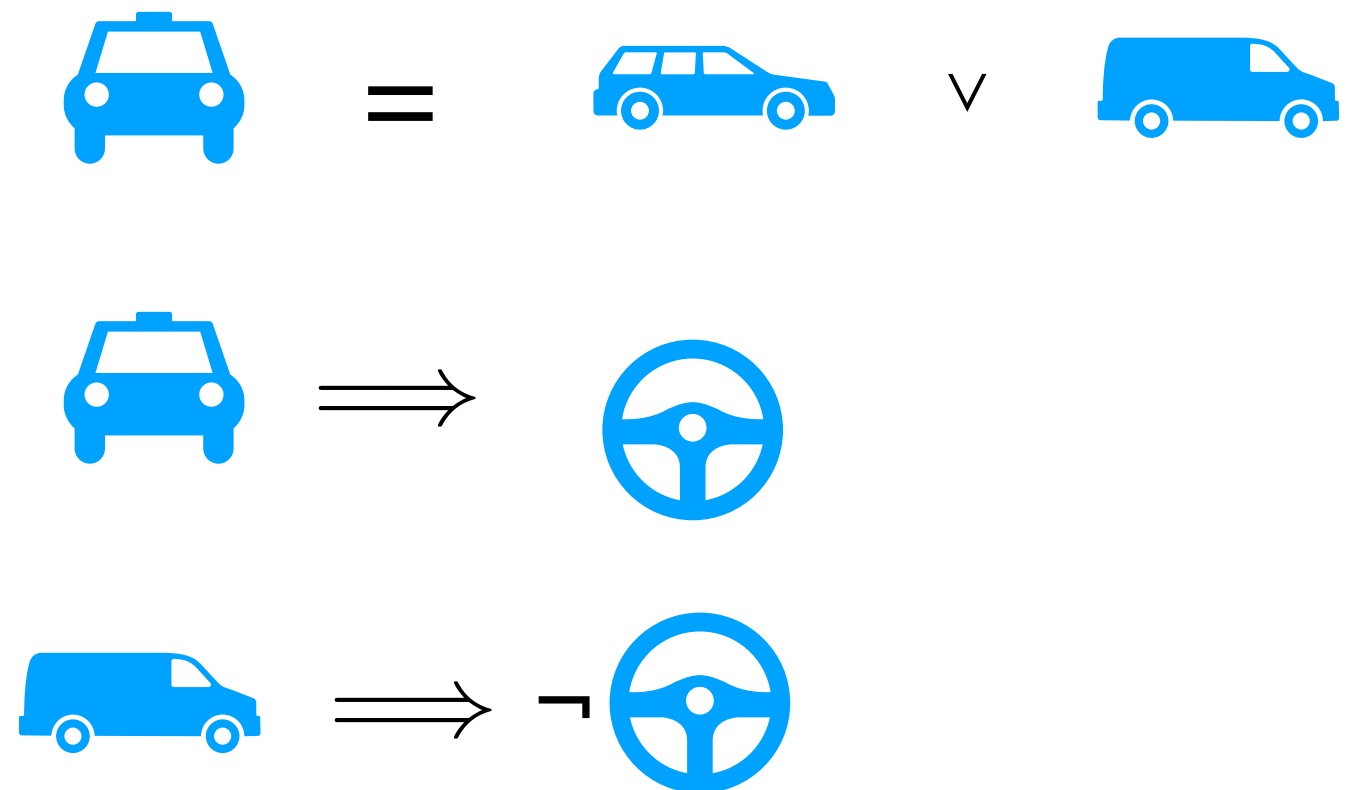
Fails for partial models!

How lazy do you like your formalisation?

make sure that the rules on paper
and in Isabelle are the same

$\text{OCDCL}_W = \text{OCDCL} + \text{restrictions}$

Another application: Dead features



Can every option be true?

How lazy do you like your formalisation?

Christoph's view:

$\text{CDCLcm}_W = \text{CDCL} + \text{improve} + \text{conflict rules}$

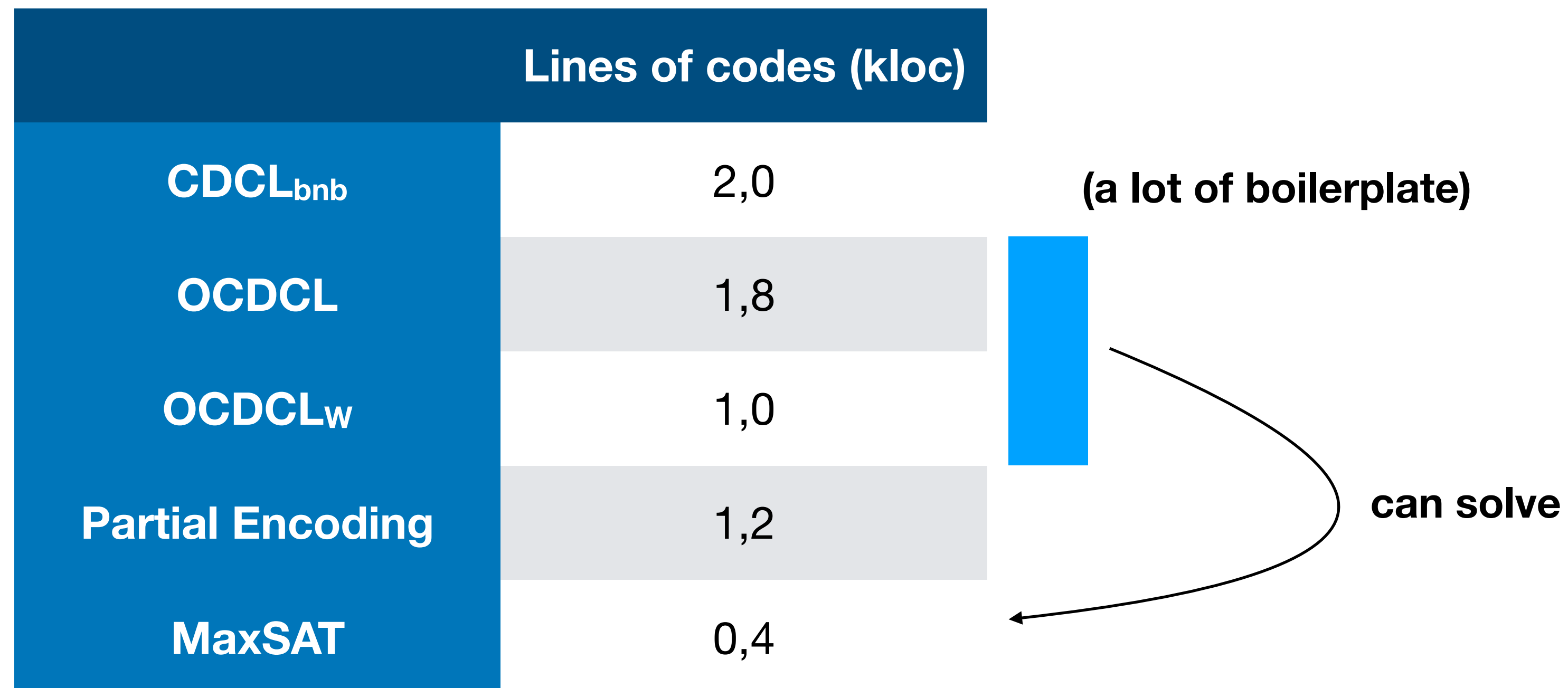
copy-paste of proofs

My idea:

$\text{CDCLcm} = \text{CDCL}_{\text{bnb}}$ where

$\mathcal{T}(\text{models_founds}) = \{-M. \text{ there is a model with more trues in models_founds}\}$

How lazy do you like your formalisation?



Conclusion

Concrete outcome

• CDCL with branch and bound

OCDCL = CDCL_{bnb} where

$$\mathcal{T}(\text{min_cost}) = \{-M. \text{cost } M \geq \text{min_cost}\}$$

OCDCL = CDCL_{bnb} where

$$\mathcal{T}(\text{min_cost}) = \{-D. \{M. \text{cost } M \geq \text{min_cost}\} \neq D\}$$

Future work

▶ CDCL(\mathcal{T})

Conclusion: How about CDCL(\mathcal{T})?

But isn't CDCL(\mathcal{T}) exactly:

CDCL_{bnb} where

$\mathcal{T} = \{\text{clauses entailed theory}\}$

Not exactly, because the wrong conflict clause (negation of the trail) is used

Translate to reuse

Theory propagation

Propagate rule

$$\begin{aligned} C \vee L \in N + \mathcal{T}(\text{min_cost}) &\implies M \models_{\text{as}} \neg C \implies \\ \text{undefined_lit } M \ L &\implies \\ (M, N + \mathcal{T}(\text{min_cost}), 0) &\implies_{\text{CDCLbnb}} \\ (L \# M, N + \mathcal{T}(\text{min_cost}), 0) & \end{aligned}$$

in Isabelle